Consider the following two flows:

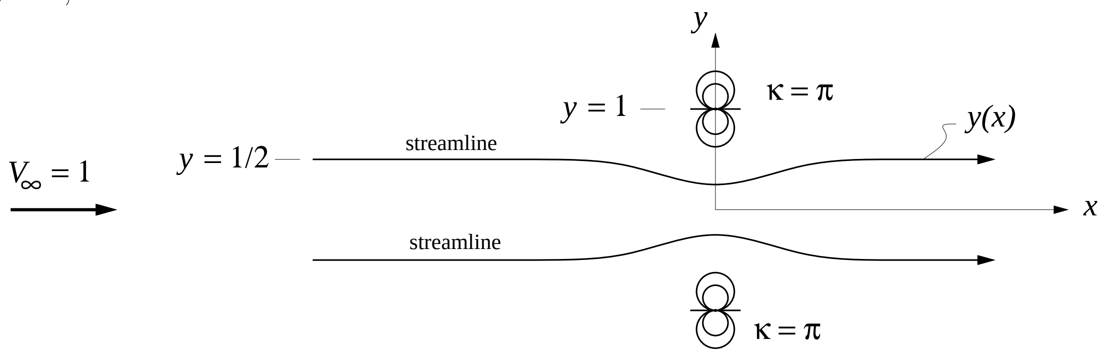
Flow A: $\psi = 2y - x^2$
Flow B: $\phi = 2x + y^2$

1a) Which flow(s) are irrotational?

1b) Which flow(s) have zero divergence?

1c) Which flow(s) would be impossible to create in a low-speed aero lab?

We wish to build a 2D channel which will contain incompressible, irrotational flow. Most of the channel will have a height=1 and velocity=1, except at a local constriction.
The constriction shape is "designed" by forming a model flow which is the superposition of a freestream of speed $V_\infty = 1$, and two doublets each having strength $\kappa = \pi$ and placed at $x, y = 0, \pm 1$ as shown.

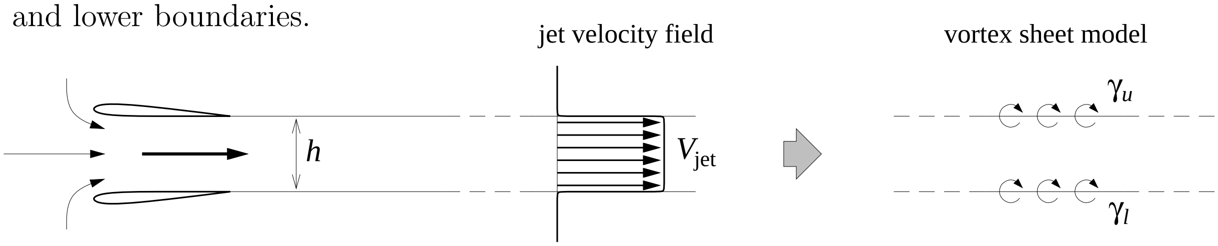


2a) We now build the real channel based on the freestream+doublet model flow, with the wall shapes matching the two streamlines which pass through $y = \pm 1/2$ far upstream. Explain why the model flow correctly represents the actual flow in this channel.

2b) Explicitly determine the velocity field of this model/real flow. You may write down an expression for $\phi(x, y)$, or $\psi(x, y)$, or $\vec{V}(x, y)$ — your choice.

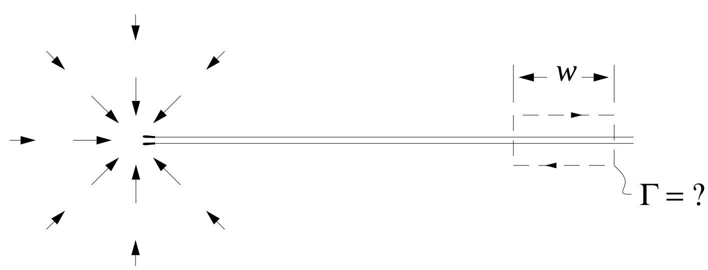
2c) Determine an equation of the form $f(x, y) = 0$ which *implicitly* defines the upper-wall shape $y(x)$. (Do not explicitly solve for $y(x)$ — that would have to be done numerically)

A stationary 2D ducted fan engine ejects air as a jet layer at some speed V_{jet}, with height h. Far downstream from the engine, the jet is simply a layer of air moving through the still atmosphere. We now wish to represent this far-downstream velocity field of the jet and surrounding still air by superimposing two very long vortex sheets placed on the jet's upper and lower boundaries.



3a) Determine the upper and lower sheet strengths γ_u and γ_l so that the resulting velocity field is the same as the actual jet flow. Note that γ is defined positive clockwise.

3b) The engine gathers air from all around. So from very far away, the slight velocity field set up by the engine must look like a 2D sink. From mass-conservation and volume-flow considerations, determine the strength Λ of this apparent sink.



3c) Determine the clockwise circulation Γ about a circuit of length w which encloses the jet as shown. Either compute it, or state the answer and justify it briefly.